Design of Missile-Aircraft Component White Noise Isolators

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Relationships are given from which the optimum design of missile-aircraft component isolators can be specified from white noise vibration. These relationships are derived from theories of linear viscoelasticity, vibration, heat conduction, and statistics of extremes. A Voigt model is used to demonstrate the thesis. A design format is given which relates mission time, environmental temperature, white noise intensity level, reliability, weight of the isolated component to rattlespace, isolator thickness, and the effective stressed area of an isolator all as a function of storage modulus, loss tangent (damping), melt temperature, and thermal conductivity of a polymeric isolator. An example calculation demonstrates the procedure that can be used for specifying an optimum vibration isolator to suit a given mission.

Nomenclature

a = effective radius of isolator, in.
a_T = ratio of relaxation times at two different temperatures
c = dashpot damping coefficient, lb-sec/in.^{2}
D = specific damping energy, in-lb/in.^{3}-cycle
d = isolator spacing, in.
E = Young's modulus, lb/in.^{2}
E_1 = normal storage modulus, lb/in.^{2}
E_2 = normal loss modulus, lb/in.^{2}
f_n = undamped natural frequency of component-isolator, Hz
G = $n\pi a^2$ = effective cross sectional area of isolator, in.^{2}
g = acceleration due to gravity, 32.2 ft/sec^2
J = coefficient relating D to σ_{max}, in.^{2}/lb-cycle
K = thermal conductivity, Btu/hr-ft°F
k = isolator spring stiffness, lb/in.
m = mass of the component to be isolated, slugs
N = number of cycles to failure
n = number of load-carrying elements
n_a = average number of zero crossings per unit time
$P(\tau)$ = probability that exceedance occurs (i.e., $y_{rel} \geq y_1$) before time τ
S = isolator strain energy, in.-lb/in^3
T = temperature, °F or K
T_s = temperature of isolator surface, °F
T_m = melt temperature of isolator, °F
T_{max} = maximum temperature above T_s attained by isolator due to vibration, °F
T_o = reference temperature of a_T, K
T_{ref} = temperature at which fatigue data is obtained, °F
t = time, sec
W = weight of missile/aircraft component, lb
W_a = acceleration power spectral density, g^2/Hz
x = cushion thickness of isolator, in.
x_{min} = minimum cushion thickness of isolator, in.
y_1 = maximum allowable displacement rattlespace of component with respect to platform, in.
$y_i(t)$ = time-dependent platform displacement, in.
$y_{rel}(t)$ = time-dependent component displacement with reference platform, in.
$\dot{y}_{rel}(t)$ = time-dependent component velocity with reference to platform, in./sec.
z = particular value of the standardized, or normalized, statistical variate
β = isolator damping
$\tan \delta$ = loss tangent
ϵ = isolator strain
ϵ_{max} = maximum design isolator strain, in./in.
σ_s = static stress in isolator, lb/in.^{2}
σ_{max} = maximum isolator stress, lb/in.^{2}
τ = the actual mission time to first exceedance (i.e., when $y_{rel} = y_1$), sec
ω = angular frequency, rad/sec
χ = generalized response variable

Introduction

THE method of isolating a missile/aircraft component from the operational and physical environment is to place an energy conversion or shielding material between the component and the platform that is part of the missile/aircraft primary supporting structure. Care must be used in the selection of a material and design configuration of an isolator since space and weight are at a premium for flight vehicles.[1] Design practices are well established and have been documented in various monographs.[2-5]

As a practical matter, isolators are chosen by designers for either a shock or vibration isolation mode by estimating relationships among displacements which determine rattlespace, loads which are to be isolated, and excitation frequencies using the Voigt model (parallel spring and dashpot) shown in Fig. 1. An isolator cushion, x thick, may consist of a number, n, of load carrying elements of radius, a, which are equally separated by a distant, d, as shown in Fig. 1. The temperature dependent mechanical properties of the cushion are approximated by the spring stiffness, k, and the damping coefficient, c, of the Voigt model. The problem is to limit the motion $y_{rel}(t)$, of the component of mass, m, to a specified rattlespace, $\pm y_1$, when the rigid platform experiences a motion $y_i(t)$. Commercial catalogs are then used to identify a reasonably suitable isolator. Applicability of the candidate isolator is then verified by test. Unfortunately such a procedure does not insure minimum weight and volume. Therefore, there is incentive to introduce a method by which designers can specify both material and design requirements to suit a specific mission requirement.

Further sophistication of isolator design serves little purpose since the environment defined in a contract specification may not be realistic. Current effort by the Navy has been to completely examine the problem of identifying the military environment. As a result of such effort military environmental specifications are being subjected to significant evaluations and revisions are envisioned.

There is reason to believe that the vibration aspect of the flight environment is nondeterministic and can be better approximated as being random instead of a well-defined spectrum of particular frequencies. Hence, it would be useful to develop a simple analysis that a designer can use to relate an approximation of random vibration (i.e., white noise) excitation to the physical properties of a polymeric isolator subjected to geometric design constraints. By this manner of approximation the thermal and mechanical properties, subject to a given design configuration, can be rationally specified for a particular application.

The paper first discusses white noise excitation and characterizes it in terms of the statistics of extremes.[6] Next the thermal and mechanical properties of linear viscoelastic polymers[7] are related to each other using a Voigt model, a failure criteria, and the concepts of linear elasticity,[8] vibration,[2] and heat conduction theories.[9] These

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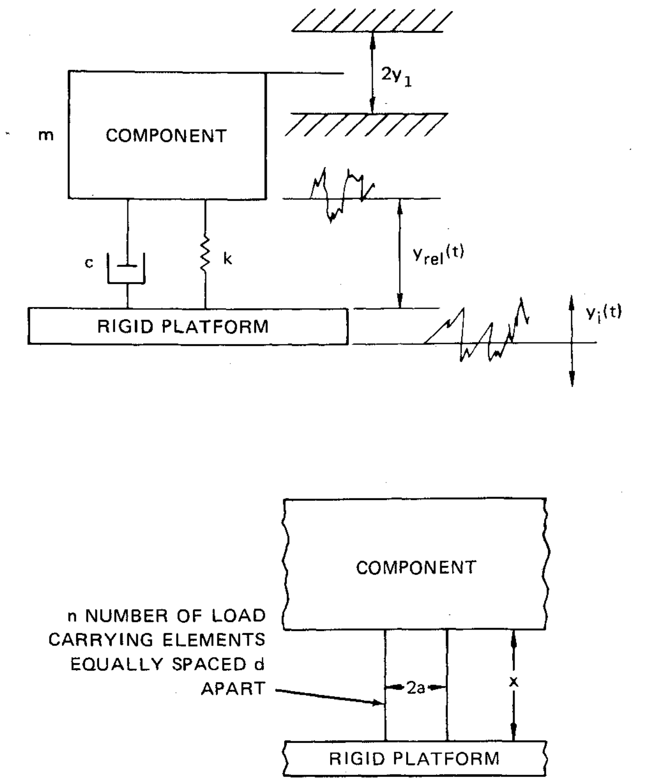

Fig. 1 Voigt model and geometric configuration representation of isolator.

concepts are then all related to each other by noting the physical significance of the standardized variate. Finally, an example design problem is illustrated to demonstrate the proposed analysis.

Isolator Model Representation

An aircraft/missile component can be isolated from the transient thermal and mechanical energies intrinsic to the platform and the environment by selecting an isolator that will absorb, redistribute, and reflect these energies away from the component. The approach taken in this study to approximate the mechanisms of such phenomena will be to assume that the linear versions of the theories of elasticity, viscoelasticity, vibration, heat conduction, and the statistics of the extremes are applicable.

The single-degree-of-freedom lumped parameter Voigt model shown in Fig. 1 will be used in this study to illustrate the thesis. This model is chosen because of its usefulness in linear isolator theory. The model relates the physical properties of an isolator to the motions and forces on the component as a function of the environment. The mass, m, represents the component to be isolated from the platform and environment. The rigid platform is assumed to be subjected to a broad band white noise induced motion $y_i(t)$. The low damped isolator filters a relatively narrow band of the white noise and induces a component motion $y_{rel}(t)$.

The design goal is to minimize the total thickness and weight of the isolator. Since the thermal energy derived from isolator hysteresis and the environment can be isolated or controlled by tailoring the insulation and thermal-mechanical properties of the isolator, the objective is to determine the relationship of the spring constant (k) and dashpot constant (c) to the isolator geometry and environmental loading in terms of the thermal and mechanical properties of the isolator.

White Noise

Random vibration is a complicated time-varying function that is characterized by irregular variations in amplitude and frequency. A recent monograph[4] states that no encompassing methodologies are available for optimizing realistic isolation systems under general random environments. Therefore, an approximation of random vibration is needed to reduce experimental data to a format useful for isolator design.

A spectral representation of random input is sufficient for the design of linear isolators. In essence a spectral density is defined as an integral of an auto correlation function which itself is obtained from an integral of a time varying motion which, for our purposes, must be assumed to be a stationary ergodic random function of distance. A random process whose power spectral density is constant is said to be white noise, which implies a wide-band disturbance. For wide-band random vibration excitation of a lightly damped isolation system the response is essentially sinusoidal vibration with a randomly varying amplitude and a statistical average frequency corresponding to the isolation system resonant frequency. The magnitude of random vibration excitation or response cannot be specified uniquely at any given instant of time. Therefore, random vibration is characterized by its rms magnitude, its power spectral density, and the probability distribution of its magnitude.

White noise excitation of an isolator simply means that the excitation at all frequencies has an equal power spectral density. Since an isolator is narrow-band (acts as a filter) it is only necessary that the excitation power spectral density curve be only approximately flat in this band region to give a close approximation to predicting the response of a simple isolator such as the Voigt model.

It has been shown[3] that the variance of the relative motion $y_{rel}(t)$ can be expressed in terms of its damping (β), white noise input acceleration spectral density (W_a), and undamped natural frequency (f_n) of the isolator which exhibits light damping ($\beta < 0.1$) by

$$\overline{y^2_{rel}(t)} = 150.1(W_a / 2\beta f_n^3) \tag{1}$$

The error using Eq. (1) for broad-band excitation at the platform (Fig. 1) with narrow-band response at the component (Fig. 1) is negligible when the upper frequency limit of integration is greater than four times $f_n/2\beta$.

Using statistics of the extremes[3,6] the actual mission time (τ) when $|y_{rel}(t)|$ will first exceed the distance, y_1, can be estimated. If z is the maximum value of the standardized variate, i.e.,

$$z = (y_1 / (\overline{y^2_{rel}(t)})^{0.5} \tag{2}$$

it can be shown[3,6] that for $z > 3.0$

$$z = \left\{2 \log \frac{\tau n_0}{-\log[1 - P(\tau)]}\right\}^{0.5} \tag{3}$$

where for white noise the average number of zero crossings ($y_{rel} = 0$) per unit time is given by

$$n_0 = 2f_n \tag{4}$$

and $P(\tau)$ is the probability that exceedance occurs before or at time τ.

Equations (1-4) relate motion, time of motion, and isolator physical properties to the probability that a given rattlespace will be exceeded.

Thermal-Mechanical Characterization of Polymers

A solid polymer experiencing sinusoidal loading (or nonsinusoidal disturbances using Fourier's rules) can be characterized in a constant temperature field by two viscoelastic moduli. One modulus is called the storage modulus (E_1) and is defined in terms of the quasi-static uniaxial maximum stress (σ_{max}), the maximum strain (ϵ_{max}) and the phase angle (δ) between the instantaneous stress and strain by

$$E_1 = \left(\frac{\sigma_{max}}{\epsilon_{max}}\right)\cos\delta \quad (5)$$

The second modulus is called the loss modulus E_2 and is defined by

$$E_2 = \left(\frac{\sigma_{max}}{\epsilon_{max}}\right)\sin\delta \quad (6)$$

It is convenient to divide Eq. (6) by Eq. (5) to obtain the loss tangent ($\tan\delta$) for a polymer as

$$\tan\delta = E_2/E_1 \quad (7)$$

The usefulness of the loss tangent is that it can be readily measured.

The physical significance of E_1, E_2, and $\tan\delta$ are that they are a measure of the elastic potential energy, of the energy dissipated to heat, and of the ratio of energy stored to energy lost, respectively, per cycle at a given frequency (ω) and temperature (T). Experiment shows that for glassy, crystalline or lightly cross-linked polymers that $\tan\delta < 0.1$. For such small values of δ, it is apparent from Eq. (5) that E_1 can be approximated as being equal to Young's modulus (E) of classical linear elastic theory.

If it is assumed that the equivalent time scale (t) for a constant strain rate uniaxial tensile/compression test is equivalent to the reciprocal of the frequency (ω) of a sinusoidal test, then E_1 can be approximate by E obtained from a uniaxial test if the strain rate is such that the tensile/compression stress (σ) or strain (ϵ) equals the magnitude of σ_{max} or ϵ_{max}, respectively, from the sinusoidal test at a time ($t = 1/\omega$). For example, if an isolator is to function at $\epsilon_{max} = 10\%$ and $\omega = 100$ rad/sec, then candidate materials can be identified by subjecting them to a uniaxial experiment with a strain rate ($\dot{\epsilon}$) equal to $(0.1)(100) = 10$ in./in.-sec and measuring the associated $E \sim E_1$ at the ten percent strain level.

An advantage of using the concept of E_1, E_2, or $\tan\delta$ is that the mathematics associated with linear viscoelastic material properties (which is used to approximate real material behavior) corresponds with the mathematics associated with linear elastic material properties (the correspondence principle). The single degree equation of motion is given by (see Fig. 1)

$$\ddot{\chi} + 2\beta\omega_n\dot{\chi} + \omega_n^2\chi = \omega_n^2\phi(t) \quad (8)$$

where $\beta = c/2(km)^{1/2}$ is the damping ratio, $\omega_n = (k/m)^{1/2}$ is the undamped natural frequency, χ = the generalized response variable, and $\phi(t)$ = the generalized, time-dependent, excitation variable. By applying the correspondence principle to Eqs. (5-7), they can be rewritten in terms of the Voigt model parameters at a given environmental temperature (T_s) in terms of the isolator thickness (x) and cross-sectional area G by

$$E_1 = E(\omega) = k(\omega)x/G \quad (5a)$$

$$E_2 = \omega c(\omega)x/G \quad (6a)$$

$$\tan\delta = \omega c(\omega)/k \quad (7a)$$

Note that Eq. (7a) at $\omega = \omega_n$ becomes

$$\tan\delta = 2\beta \quad (9)$$

It is noted that E_1, E_2, and $\tan\delta$ can also be calculated if the mechanical impedance parameters of a given isolator have been measured.

It is well known for polymers in general that E_1, E_2, and $\tan\delta$ are functions of temperature as well as frequency. By assuming that the linear viscoelastic model representation of a real material is also thermorheologically simple,[10] a great simplification is introduced for obtaining design material property data. Such simplification is useful for establishing design tradeoffs as a function of operational and environmental parameters. In essence E_1 and E_2 are made functions of a reference temperature (T_o) and the log ωa_T, where a_T is called the time-temperature shift factor and is solely a function of T. (It is assumed that the temperature field in the isolator is uniform at any instant of time.) In this manner measuring E_1 or E_2 at some ω and T is equivalent to measuring E_1 or E_2 at frequency ωa_T and T_o.

The amount of energy dissipated in the isolator per cycle is given by

$$D = \int_0^{2\pi/\omega} \sigma\frac{d\epsilon}{dt}dt \quad (10)$$

The expression for sinusoidal strain is

$$\epsilon = \epsilon_{max}\sin\omega t \quad (11)$$

and sinusoidal stress which lags the strain by δ is

$$\sigma = \sigma_{max}\sin(\omega t + \delta) \quad (12)$$

Substituting Eqs. (11) and (12) into (10) and integrating yields

$$D = \pi E_2\epsilon_{max}^2 \quad (13)$$

Since $E_1 \sim E = \sigma/\epsilon$, substitution into Eq. (13) together with the relationship of Eq. (7) yields

$$D = \frac{\pi}{E_1}\sigma_{max}^2\tan\delta = J\sigma_{max}^2 \quad (14)$$

Typical values for J are between 10^{-4} and 10^{-2}. For preliminary design, J can be taken as 10^{-3}. The mechanical energy dissipation predicted by Eq. (14) can be assumed to be converted into heat energy which can be entirely characterized by the thermal diffusivity of the isolator. Since polymers are poor conductors, the temperature within the isolator will rise until a thermal energy balance is established at the isolator boundaries. Obviously, a rise in temperature will significantly change the mechanical properties.

An estimate of an average maximum temperature (T_{max}) of the load carrying elements (Fig. 1) is made by assuming that each element experiences homogeneous sinusoidal stress fields and hence a constant rate of heat production D per unit time per unit volume. It is further assumed that the heat flow in the load carrying element is radial and that its surface temperature is maintained constant. The solution to this heat transfer problem is known[9] and results in the prediction of a radial temperature gradient in the load-carrying element. Since isolators are chosen on the basis that their damping ($\tan\delta$) is largest at the undamped natural frequency (f_n), the highest temperature rise occurs at resonance. It remains to choose T_{max} from the predicted radial temperature gradient in the load carrying element. The value of the temperature at $a/2$ is taken as an approximation for T_{max} and is given[9] at steady state by

$$T_{max} = 0.868\,Da^2f_n/K \quad (15)$$

where K is the thermal conductivity of the polymer. For the Voigt model, $f_n = (k/m)^{0.5}/2\pi$. Substituting f_n into Eq. (5a) noting that $m = W/g$ and taking G as the effective stressed area gives

$$f_n = 3.13\left(\frac{E_1G}{Wx}\right)^{0.5} \quad (16)$$

Substituting Eqs. (14) and (16) into (15) gives

$$T_{max} = 2.71\frac{a^2J}{K}\sigma_{max}^2\left(\frac{E_1G}{Wx}\right)^{0.5} = \frac{2.71\pi a^2\sigma_{max}^2}{E_1K}\left(\frac{E_1G}{Wx}\right)^{0.5}\tan\delta \quad (17)$$

Equation (17) gives the relationship among the design variables a, G, x, the mechanical properties E_1, $\tan \delta$, the thermal property K, the maximum allowable temperature T_{max} and stress σ_{max} needed to isolate a component of weight W. The upper limit of T_{max} is determined by the chemical stability of the polymer whereas the upper limit of σ_{max} is determined by a suitable failure stress criteria.

The failure mechanism envisioned for the vibration isolator is fatigue whereas for shock its the ultimate tensile strength. If appropriate fatigue data is not available, a conservative approximation can be made at T_{ref} = 80°F for low modulus polymeric materials as

$$N = 10^8 - 6.6 \times 10^5 S \tag{18}$$

where N is the number of cycles and S is the corresponding strain energy. For a low damped and linear material $S = \sigma_{max}\epsilon_{max}/2$, but $\epsilon_{max} = \sigma_{max}/E_1$, and therefore

$$N = 10^8 - 3.3 \times 10^5 \frac{\sigma_{max}{}^2}{E_1} \tag{19}$$

Equation (19) suggests that the maximum number of cycles that a low modulus polymeric material can be expected to survive as $\sigma_{max} \rightarrow 0$ is some 10^8 cycles.

The kinetic theory of elastomers states that stress is proportional to absolute temperature. Therefore, it is proposed until experimental data for a given candidate polymer is available that Eq. (19) be modified to account for $T_{ref} \neq$ 80°F by multiplying the right-hand side by the factor $(T_m - [T_s + T_{max}])/(T_m - T_{ref})$. T_m is the polymer melt temperature which can be assumed to be 200°F if not otherwise known. T_s is the surface temperature of the load carrying element. T_{ref} is the temperature at which the fatigue data are obtained. The temperature compensated version of Eq. (19) becomes

$$N = \left[10^8 - 3.3 \times 10^5 \frac{\sigma_{max}{}^2}{E_1}\right] \frac{(T_m - [T_s + T_{max}])}{(T_m - T_{ref})} \tag{20}$$

This temperature correction predicts that N decreases with increasing isolator operating temperature (T_{max}) and that the fatigue life at −50°F for a given σ_{max} is about double that at +80°F. Equation 20 states that a temperature (T_{max}) rise in the isolator due to vibration at resonance lowers its lifetime. The worst case is obviously for $T_s + T_{max} = T_m$, recognizing, of course, that it takes time to reach T_{max} (a function of the thermal diffusivity of the isolator).

The failure criteria based on Eq. 18 is based on sinusoidal loading. To apply Eq. (18) to random vibration loading is known to be conservative.[11] The degree of conservatism for a particular isolator material and design must be evaluated from specific experimental data.

Design Equations

The amount of creep/relaxation acceptable in an isolator depends largely upon storage requirements. In general, storage strains and stresses in filled elastomers should be less than 5% and 1.0 lb/in.², respectively, when storage periods at +80°F for over a year are envisioned. The amount of viscoelastic phenomena tolerated in the isolator can be traded off for reliability and/or allowable motion transmissions to the component for a given design and mission.

In order to incorporate specific design tradeoff capability into the Voigt model, it will be assumed that c and k are composed of a number (n) of separate elements with equal properties. More complex conceptual designs can be related to this baseline design by using foam technology or treating symmetrical segments of a complex design as being analogous to complex components (incorporating shear and/or coulomb energy dissipation mechanisms). The isolator design parameters are the isolator thickness

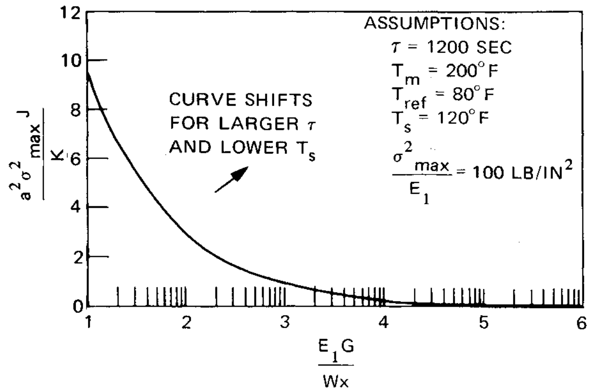

Fig. 2 Relationship between $a^2\sigma_{max}{}^2J/K$ and E_1G/Wx given a set of material assumptions.

(x), rattlespace y_1 and the average radius (a) of each separate isolator.

The component weight (W) will induce a static stress (σ_s) equal to W/G where G is the "effective" total cross-sectional area of the isolator (i.e., $\approx \pi n a^2$). The storage stress design requirement is satisfied if $G \geq W$. It follows that for a given x, minimum isolator weight is achieved for $G = W$.

It remains, then, to determine the design criteria necessary for specifying an efficient isolator. The objective is to determine the largest acceptable isolator stress (σ_{max}) induced by the platform (Fig. 1) by selecting an isolator material with mechanical moduli small enough to minimize transmittance, yet large enough to allow noncreep storage. σ_{max} can be lowered by increasing G, but this will increase the isolator weight. The procedure for choosing a candidate isolator material involves the selection of a polymer of the desired mechanical moduli and then to determine σ_{max} on the basis of the fatigue failure strength. Simultaneously thermal energy (either created within the isolator or from the platform) can be isolated from the component by tailoring the thermal and mechanical properties. Note that isolation of the component from thermal and mechanical energies is achieved by allowing the component to move (y_{rel}) with respect to the platform induced motions (y_i). The objective is then to determine the relationship of the spring constant (k) and dashpot constant (c) to the isolator geometry and environment motion in terms of the thermal and mechanical properties of the isolator.

A lightly damped narrow-band isolator system exposed to broad-band excitation exhibits behavior similar to that of a general system exposed to white noise. Therefore on the average each cycle (N) of the responding system (component) is characterized by two zero crossings. Therefore Eq. (4) can be used to estimate N for a mission time τ by

$$N = f_n \tau \tag{21}$$

Substituting Eq. (17) into (20) as well as the value for N obtained by substituting Eq. (16) into (21) results in

$$3.13\tau\left(\frac{E_1G}{Wx}\right)^{0.5}(T_m - T_{ref}) = \left[10^8 - 3.3 \times 10^5 \frac{\sigma_{max}{}^2}{E_1}\right] \times \left[T_m - T_s - \frac{2.71a^2J}{K}\sigma_{max}{}^2\left(\frac{E_1G}{Wx}\right)^{0.5}\right] \tag{22}$$

The usefulness of Eq. (22) as a design tradeoff basis can be demonstrated for a class of low modulus elastomers, i.e., elastomers for which $\sigma_{max}{}^2/E_1 <$ 100 in.-lb/in.³. By assuming τ = 1,200 sec, T_m = +200°F, T_{ref} = +80°F, and T_s = 120°F, Eq. (22) is used to relate $a^2J\sigma_{max}{}^2/K$ to E_1G/Wx for $\sigma_{max}{}^2/E_1$ = 100 in.-lb/in.³ as shown in Fig. 2. For lower T_s and longer τ, the curve shifts up and to the right

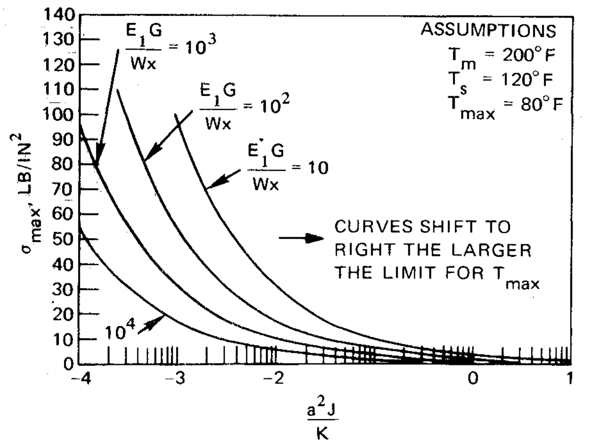

Fig. 3 Relationship between σ_{max} and a^2J/K as a function of E_1G/Wx given a set of material and operation assumptions.

and is less critical for design considerations. Figure 2 illustrates that for a given isolator material and component weight to minimize x requires that $a^2\sigma_{max}^2$ be minimized.

Further insight into the relationship between σ_{max} and (a) is obtained for the condition of negligible self heating, or in the given example for $T_m - T_s = +80$°F. It follows that the relationship

$$2.71(a^2J\sigma_{max}^2/K)(E_1G/Wx)^{0.5} = 80°\text{F} \qquad (23)$$

satisfies the conditions for fatigue failure at the largest acceptable value for σ_{max}. Figure 3 illustrates the relationship given by Eq. (23) among the largest acceptable σ_{max}, a^2J/K, and E_1G/Wx. Thus, if x_{min} is determined by other considerations such as storage or shock, and σ_{max} is obtained experimentally, then (a) can be taken directly from Fig. 3, given an isolator material and component weight.

A physical interpretation of z can be made by noting that $y_1 \sim \epsilon_{max}\, x$, and $\overline{y_{rel}^2}\,^{0.5}$ is simply a "standard" displacement approximated by ϵx, hence $z = \epsilon_{max}/\epsilon$. It follows that the larger the value for z, the smaller the operational strain ϵ for a given material ϵ_{max}.

For a given isolator design, suppose τ = 1,200 sec and T_s = +120°F. $P(\tau)$ is the probability that $y_{rel} > y_1$ occurs before time, τ, and is assigned with a low risk factor, e.g., $P(\tau) = 0.001$. It then follows that when exposure time and space availability are fixed, the designer must control the

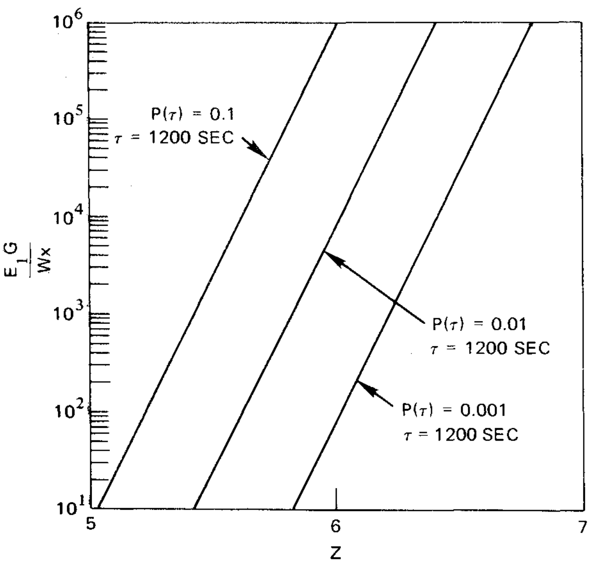

Fig. 4 Relationship between E_1G/Wx and z for various $P(\tau)$ and τ.

natural frequency and damping of the isolator if $y_{rel} \le y_1$ with the stipulated probability.

Combining Eqs. (1–4) substituting for β and f_n from Eqs. (9) and (16) and for illustration purposes assigning a large value for white noise excitation (W_a = 0.1 g^2/Hz); results in the requirement to satisfy the following two relationships:

$$2\log\left(\frac{6.26\tau\left(\frac{E_1G}{Wx}\right)^{0.5}}{-\log[1-P(\tau)]}\right) \ge 9(=z^2) \qquad (24)$$

and

$$2.04y_1^2\tan\delta\left(\frac{E_1G}{Wx}\right)^{1.5} \ge 9(=z^2) \qquad (25)$$

Equation (24) is solved for E_1G/Wx as a function of z with the parameters $P(\tau)$ and τ and is shown in Fig. 4. Note from Fig. 3 that for a given a^2J/K, that σ_{max} can be made larger if the magnitude of E_1G/Wx is reduced. The design goal is to choose the smallest value for E_1G/Wx so that the maximum σ_{max} is used for a particular isolator design. From Fig. 4 it is observed that for a given E_1G/Wx, the requirement is only that z be increased by 0.4 to lower $P(\tau)$ from 0.1 to 0.001.

Figure 4 shows that the larger the probability for exceeding, the smaller the value for z, and the larger the operational strain ϵ for a given ϵ_{max}. The most conservative design is to incorporate the largest value of z together with the smallest $P(\tau)$. The more conservative the design, the thicker the isolator, hence, the higher the consequential isolator weight. Since safety is already considered statistically through the probability factor, the design goal is to minimize both z and E_1G/Wx.

The value of a minimum y_1 that should be used in Eq. (25) may be determined by the shock mitigation or storage requirements. For illustrative purposes assume a 20 g for 30 msec rectangular shock.

The analysis that can be used to obtain $(y_1)_{min}$ is the strain energy-kinetic energy equality relationship. Until experimental evidence for a particular isolator material is available, it is safe to use Eq. (18) by substituting $N = 1$ to obtain S_{max} = 150 in.-lb/in.^{3}. The total potential strain energy capacity in the isolator is then some 150 $G(x_{min})_{shock}$ in.-lb. Now the maximum velocity step of the platform reacted by the component is $\dot{y}_{rel}$ = (20 g) (386 in./sec^2-g) (0.030 sec) = 232 in./sec. It follows that the kinetic energy to be absorbed by the isolator is (232 in./sec)^2 $W/2\,g \sim 68.9\,W$. Equating the potential to kinetic energys gives the relationship

$$(x_{min})_{shock} = 0.45(W/G) \qquad (26)$$

Since y_1 is a rare event assume

$$(y_1)_{min} = (x_{min})_{shock} \qquad (27)$$

The determination of an optimum y_{max} is dependent upon the structural stability of a given isolator design. For a simple circular isolator, experience has shown that column instability will not occur if $x \le 1.2a$. A generality is that the stiffness response of an elastomeric isolator increases significantly for $\epsilon_{max} > 0.3$ in./in. because of barrelling effects resulting from the incompressible nature of elastomers. Therefore for preliminary design assume $\epsilon_{max} \le 0.3$ in./in. It remains next to illustrate the above analytical procedure by selecting the preliminary design parameters for an isolator subjected to a hypothetical specification, W_a = 0.1 g^2/Hz. Suppose a large component is to be isolated. The designer is given that W = 700 lb, $x \le$ 1.3 in., $P(\tau)$ = 0.001, T_s = 120°F, τ = 1200 sec, single lifetime shock = 20 g for 30 msec rectangular, depot storage of 1-year at 80°F and component surface area is such that $G \le 3740$ in.^{2}.

The storage requirement suggests $W/G \le 1$ lb/in.^{2},

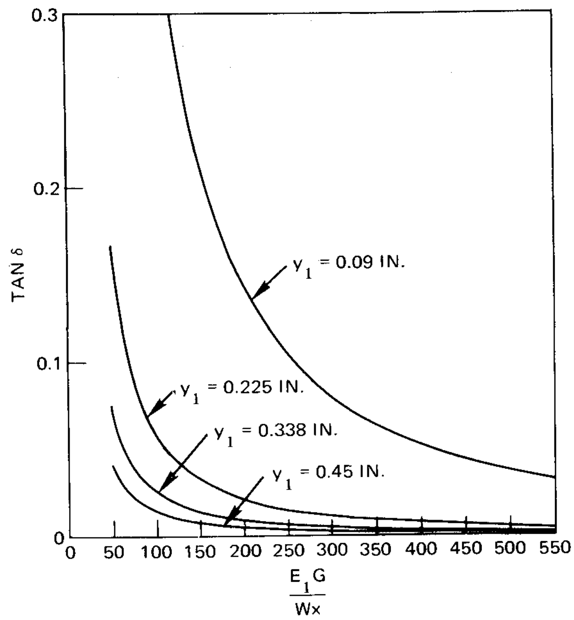

Fig. 5 Relationship between tan δ and E_1G/Wx as a function of y_1 for $z = 6.1$.

which means $G \geq 700$ in.^{2}. From the component surface area requirement it follows that $W/G \geq 0.19$ lb/in.^{2}. Taking four arbitrary values for W/G (e.g., 0.19, 0.5, 0.75, and 1 lb/in.^{2}) and substituting in Eq. 26 determines $(x_{min})_{shock}$ = 0.05, 0.23, 0.34, and 0.45 in., respectively. Since $700 \leq G \leq 3740$, begin by choosing $G = 700$ in.^{2} on the basis of minimizing weight and E_1G/Wx.

The relationship of y_1 with respect to tan δ and E_1G/Wx can be found [Eq. (25)] once bounds are placed upon E_1G/Wx. The smallest (desirable) realistic values of E_1 for candidate low-modulus elastomeric materials lies between 100 and 200 lb/in.^{2}. For a given material, E_1 is a function of both frequency and temperature and since $0.2 \leq W/G \leq 1$ lb/in.^{2} and $x_{min} = 1.3$ in., it follows that $77 \leq E_1G/Wx \leq 770$ is the range expected to be found for candidate materials. From Fig. 3, for $P(\tau) = 0.001$, it is seen that $6 \leq z \leq 6.2$. For design purposes, assume $z = \epsilon_{max}/\epsilon = 6.1$. Equation (25) is then used to relate y_1 $[=(x_{min})_{shock}]$ to both tan δ and E_1G/Wx as shown in Fig. 5. Material damping is seen to become a critical material property requirement as y_1 and E_1G/Wx get smaller. Now since $\epsilon_{max} = y_1/x$, it follows that $0.11 \leq \epsilon_{max} \leq 0.53$. For many low-modulus elastomers this range for ϵ_{max} is attainable.

If tan δ, E_1, and K are not known for a given low-modulus elastomer as a function of temperature and frequency, one assumes a typical set of values for $f_n < 2000$ Hz and $T_{ref} = +80°F$ as being tan $\delta = 0.13$, $E_1 = 150$ lb/in.^{2}, and $K = 0.11$ Btu/ft-hr.°F. Then $J = 2.7 \times 10^{-3}$ and $(E_1G/Wx)_{min} = 115$. From Fig. 2, it follows that $a\sigma_{max} = 33.2$ lb/in.^{2}. This means that for a large range of σ_{max}, there is a corresponding large range for a. For example, assume that $\sigma_{max} = E_1\,\epsilon_{max}$, then $16 < \sigma_{max} < 80$, which corresponds to $0.4 < a < 2.1$ in. The tradeoff between σ_{max} and a for a given E_1G/Wx is shown in Fig. 3. It is desirable to minimize the motion of the component so that spurious flight dynamic perturbations are minimized. This suggests that the lowest ϵ_{max} be used, which means that σ_{max} can be used to determine a candidate value for a. $(\epsilon_{max})_{min} = 0.11$; therefore, $\sigma_{max} = 16.5$ lb/in.^{2} and from Fig. 3, $a \geq 0.64$ in. However, from structural considerations, $x \leq 1.2a$; therefore, $a \geq 0.71$ in. Obviously, for the example problem choose $a = 0.71$ in. Since $G = 700$ in.$^2 = n\pi a^2$, it follows that $n = 440$, and hence based on the total available surface area of 3,740 in.^{2}, the isolator spacing (d) is 2.6 in.

Discussion

The example illustrates how to determine the approximate dimensions for an isolator given an operational environment, design constraints, and average typical values for the mechanical and thermal properties of a class of polymers. Conversely, specifications for E_1, tan δ, and K can be obtained, given the isolator dimensions, design constraints, and the operational environment. Optimization with respect to weight and volume of the isolator can be achieved by exercising the design variables for a given set of design imposed conditions.

It is observed from Eq. (1) that the variance of the relative motion (y_{rel}^2) can be decreased for a given acceleration power spectral density by choosing a material with greater damping. However, a much greater decrease is achieved by raising the natural frequency, which can be achieved by choosing a more rigid isolator material, by increasing the diameter, or by decreasing the thickness of the isolator. On the other hand, increasing the natural frequency is limited in that the standardized variate (z) will increase according to Eqs. (3 and 4), requiring that the isolator material operate at a greater maximum strain/stress. According to Eqs. (17 and 20) this means that the fatigue life time will be reduced both because of the higher operating stress and temperature. It becomes evident then that the degree of optimization that can be achieved for a given design is highly dependent on an accurate knowledge of the thermal and mechanical properties of the candidate isolator material.

Conclusion

An inverse design approach is developed by which the thermal and mechanical properties of an isolator can be traded off with the geometric (and hence volume) configuration of an isolator for a given induced random motion during a specified time. The resulting isolator design is optimized in the sense that the lifetime is determined by failure due to a history of simultaneous heating and fatigue.

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